Aryabhatta Knowledge University (AKU)

Civil Engineering

Mathematics-III

Solved Exam Paper 2019

Q2. A) Solve

$$x(y^2+z)\frac{\partial z}{\partial x} - y(x^2+z)\frac{\partial z}{\partial y} = (x^2-y^2)z$$

Answer: $\frac{dx}{x(y^2+z)} = \frac{dy}{-y(x^2+z)} = \frac{dz}{z(x^2-y^2)}$

Choose the multipliers as $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$

$$\frac{1}{x}dx + \frac{1}{y}dy + \frac{1}{z}dz = k\{\frac{1}{k}x(y^2+z) + \frac{1}{y}(k^2+z) + \frac{1}{z}(x^2+z)$$

$$\frac{1}{x}dx + \frac{1}{y}dy + \frac{1}{z}dz = k\left(y^2+z-k^2-z+x^2-y^2\right)$$

$$\frac{1}{x}dx + \frac{1}{y}dy + \frac{1}{z}dz = 0$$

On integration

$\log x + \log y + \log z = \log c_1$

$\log (xyz) = \log c_1$

$xyz = c_1$

now again choose the multipliers as x,y,-1

$xdx - ydy - dz = k[x^2(y^2 + z) - y^2(x^2 + z) - z(x^2 - y^2)]$

$xdx - ydy - dz = k[x^2y^2 + x^2z - y^2x^2 - y^2z - zx^2 + zy^2]$

$xdx - ydy - dz = 0$

on integration

$$\frac{x^2}{2} - \frac{y^2}{2} - z = c_2$$

$x^2 - y^2 - 2z = 2c_2$

Now on combining both eq. we get a general solution

$f(xyz, x^2 - y^2 - 2z) = 0$

1. Q3. State and prove Rodrigues formula .

Sol- we derive a formula for the legendre polynomials

Formula

Now proof

Let

We shall first establish that the nth derivation of u, that is u_n is a so the legendre differentiate eq.

$(1 - x^2)y'' - 2xy' + n(n+1)u = 0 - - - - - - 1$

Differ w.r. to x

$$\frac{du}{dx} = u_1 = n(x^2-1)^{n-1}.2x$$

Or

$$(x^2-1)u_1 = 2nx(x^2-1)^u$$

i.e $(x^2-1)u_1 = 2nxu$

Diff. w.r. to x again, we have

$$(x^2-1)u_2 + 2xu_1 = 2x(xu_1+u)$$

Now differ. The result in timer by applying lebuitz theorem for derivation of a product given by

$$(uv)_n = uv_n + nu_1v_{n-1} + \frac{n(n-1)}{2}u_2v_{n-2} + ...u_nv$$

$$[(x^2-1)u_2]_n + 2(xu_1)_n = 2n(xv_1)_n + 2_xu_n$$

$$\left[(x^2-1)u_{n+2} + x2 * u_{x+1} + \frac{n(n-1)}{2}.2u_x\right] + 2(xu_{x+1} + xcu)$$

$$(x^2-1)u_{n+2} + 2n * u_{n+1} + (n^2-n)u_n + 2 * u_{n+1} + 2_nu_x = 2nxu_{n+1} + 2n^2u_x + 2$$

$$(x^2-1)u_{n+2} + 2xu_{n+1} - n^2u_n - nu_x = 0$$

Or

$$(1-x^2)u_{n+2} - 2 * u_{n+1} + n(n+1)un = 0$$

This can be put in the form

$$(1-x^2)u_x`` - 2xu_n` + n(n+1)un = 0 - - - 2$$

Comparing 2 with 1 we conclude that u_n is a solution of the legendr eq. it may be observed that U is a polynomial of degrees 2x & hence will be a polynomial of degree x.

also $p_x(x)$ which satisfies the legendre differentiate eq. is also polynomial of degree x.

$$p_n(x) = ku_n = k\left[(x^2-1)^n\right]_n$$

$$p_n(x) = k\left[(k-1)^n(x+1)^n\right]$$

Applying Leibnitz theorem for the RHS we have

$$p_n(x) = k[(k-1)^n(x+1)^n]_n + n.n(x-1)^{n-1}[(x+1)^n]_{n-1} + \frac{n(n-1)}{2}n(n-$$
$$(x-1)^{n-2}[(x-1)^n]_{n-2} + -----[(x-1)^n]_n$$

It should be observed that if $z = (x-1)^n$

$z_1 = n(x-1)^{n-1} \& z_2 = n(n-1)(x-1)^{n-2}$etc.

$z_n = n(n-1)(n-2)....2.1(x-1)^{n-1}$or

$z_n = n!(x-1)^{\circ}$

$z_n = n!$

$[(x-1)^n]_x.x!$

Putting x =1 in eq. 1 all the terms in RHS become zero except the l term which becomes $n!(1+1)^n = n!2^n$

$p_n(1) = 1$ by thedef of $p_n(x)$

$1 = \text{kn:}2^n$

$$k = \frac{1}{n!2^n}$$

$$p_n(x) = ku_n$$

$$p_n(x) = \frac{1}{2^n n!}[(x^2-1)^n]$$

$$p_n(x) = \frac{1}{2^n n!}\frac{d^n}{d^x}(x^2-1)^n$$

Q4. A coin is tossed. If it turns up H, two balls will be drawn from urn A otherwise 2 balls will be drawn from urn B. urn A contains 3 red and 5 blue balls , urn B contains 7 red and 5 blue balls. What is the probability that urn A is used , given that both balls and blue? (find in both cases, when balls were chosen with replacement and without replacement).

Sol- let us define the following events

$A_1 =$ urn A is chosen

$A_2 =$ urn B is chosen

E= two blue balls are drawn (with reputation)

Then we have

$$p(A_1),\frac{1}{2}$$

$$p(E/A) = \frac{5}{8} * \frac{5}{8}$$

$$\frac{25}{64}$$

$p(A_2),\frac{1}{2}$

$$p(E/A) = \frac{5}{12} * \frac{5}{12}$$

$$\frac{25}{144}$$

So,

$$p(A/E) = \frac{(P(A_1)\&P(E/A_1))}{P(A_1)P(E/A_1) + P(A_2)P(E/A_2)}$$

$$\frac{\left(\frac{1}{2} * \frac{25}{64}\right)}{\frac{1}{2} * \frac{25}{64} + \frac{1}{2} * \frac{25}{144}}$$

$$\frac{25}{\frac{128}{25} + \frac{25}{288}}$$

$$\frac{\frac{25}{128}}{\frac{(225+100)}{1152}} = \frac{\frac{25}{128}}{\frac{325}{1152}}$$

$$\frac{9}{13}$$

(b) for event $A_1, A_2, A_3, A_4, A_5 \ldots A_n$

$$P(\cup^n i = 1 A_i) \geq \sum_{i=1}^{n} p(A_i) - (n-1)$$

Prove that

i. $p(\cap^n i = 1) \geq 1 - \sum_{i=1}^{n} p(\bar{A}_i)$

$$p(\cap^n i = 1A_i) \geq 1 - \sum_{i=1}^{n} p(\bar{A}_i)$$

Q5. State and prove bayes theorem.

Sol. –it states that "If $A_1, A_2 - - - A_N$ are n mutually exclusive event with $P(A_1) \neq 0, i = 1,2 - - - n$ & B is any other event which can occurred with A or A_1 orA_N then we have,

$$P(A_i/B) = \frac{[P(A_i)P(B/A_i)]}{\sum_{i=1}^{n} p(A_i)P(B/A_i)}$$

Proof- by compound theorem of probability ,

We get

$$P(A_i \cap B) = P(A_i).P(B/A_i)$$

Or $P(A_i \cap B) = P(B).P(A_i/B)$

Given that , B is any other event which occur with A or A; or .....A_N i.e

$$B = B \cap (A_1 \text{or} A_i \text{or} - - - - A_n)$$

$$B \cap [A_1 \cup A_2 \bigcup - - - A_N]$$

$$(B \cap A_1) \cup (B \cap A_2) \bigcup - - - U(B \cap A_N)$$

$P[B \cap A_1 \cup (B \cap A_2) \bigcup - - - - - - P(B \cap A_n)]$

$p(B \cap A_1) + P(B \cap A_2) \pm - - P(B \cap A_n)$

$\sum_{i=1}^{n} p(B \cap A_i)$

$\sum_{i=1}^{n} p(A_i) P(B/A_i)$

Again from II

$$P(A_i/B) = \frac{[P(A_i)P(B/A_i)]}{\sum_{i-1}^{n} p(A_i)P(B/A_i)}$$

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(b) a random variable X follows binominal distribution with parameter n=40 and $p = \frac{1}{4}$ use chebyshev's inequality to find bounds for.

a. $p(|X - 10| < 8)$

b. $p(|X - 10| < 10)$