

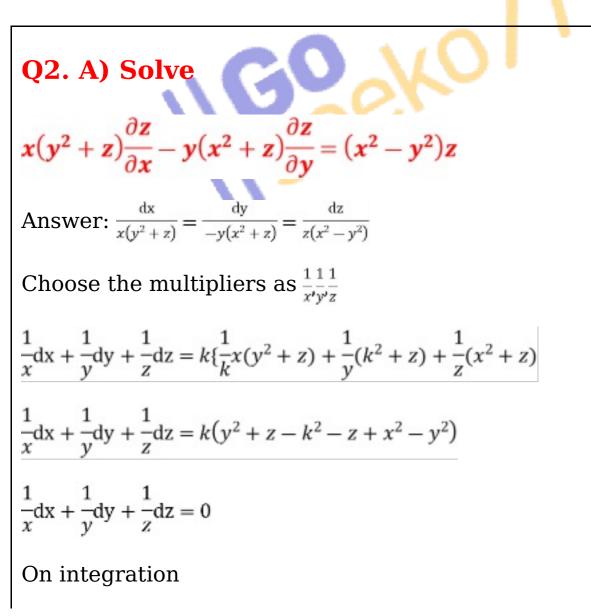


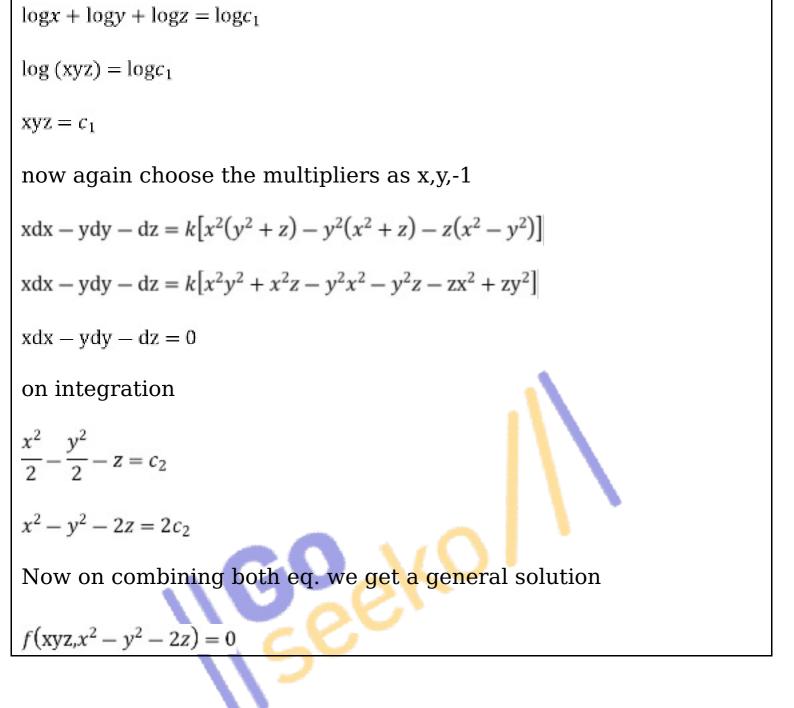
Aryabhatta Knowledge University (AKU)

Civil Engineering

Mathematics-III

Solved Exam Paper 2019





1. Q3. State and prove Rodrigues formula .

Sol- we derive a formula for the legendre polynomials

Formula

Now proof

Let

We shall first establish that the nth derivation of u, that is u_n is a so the legendre differentiate eq.

 $(1 - x^2)y^{``} - 2xy^{`} + n(n+1)u = 0 - - - - 1$

Differ w.r. to x

$$\frac{\mathrm{d}u}{\mathrm{d}x} = u_1 = n(x^2 - 1)^{n-1}.2x$$

Or

$$(x^2 - 1)u_1 = 2nx(x^2 - 1)^u$$

i.e $(x^2 - 1)u_1 = 2nxu$

Diff. w.r. to x again, we have

$$(x^2 - 1)u_2 + 2xu_1 = 2x(xu_1 + u)$$

Now differ. The result in timer by applying lebuitz theorem for derivation of a product given by

$$\begin{aligned} (uv)_n &= uv_n + nu_1v_{n-1} + \frac{n(n-1)}{2}u_2v_{n-2} + \dots u_nv \\ [(x^2 - 1)u_2]_n + 2(xu_1)_n &= 2n(xv_1)_n + 2_xu_n \\ [(x^2 - 1)u_{n+2} + x2 * u_{x+1} + \frac{n(n-1)}{2}.2u_x] + 2(xu_{x+1} + xcu) \\ (x^2 - 1)u_{n+2} + 2n * u_{n+1} + (n^2 - n)u_n + 2 * u_{n+1} + 2_nu_x &= 2nxu_{n+1} + 2n^2u_x + 2 \\ (x^2 - 1)u_{n+2} + 2xu_{n+1} - n^2u_n - nu_x &= 0 \\ Or \\ (1 - x^2)u_{n+2} - 2 * u_{n+1} + n(n+1)u_n &= 0 \\ This can be put in the form \end{aligned}$$

$$(1 - x^2)u_x = -2xu_n + n(n+1)u_n = 0 - - 2$$

Comparing 2 with 1 we conclude that u_n is a solution of the legendreq. it may be observed that U is a polynomial of degrees 2x & hence

will be a polynomial of degree x.

also $p_x(x)$ which satisfies the legendre differentiate eq. is also polynomial of degree x.

$$p_n(x) = ku_n = k[(x^2 - 1)^n]_n$$

$$p_n(x) = k[(k-1)^n(x+1)^n]$$

Applying Leibnitz theorem for the RHS we have

$$p_{n}(x) = k[(k-1)^{n}(x+1)^{n}]_{n} + n.n(x-1)^{n-1}[(x+1)^{n}]_{n-1} + \frac{n(n-1)}{2}n(n-(x-1)^{n-2}[(x-1)^{n}]_{n-2} + \dots - [(x-1)^{n}]_{n}]_{n-1}$$

It should be observed that if $z = (x-1)^{n}$
 $z_{1} = n(x-1)^{n-1} \& z_{2} = n(n-1)(x-1)^{n-2} \text{etc.}]_{n}$
 $z_{n} = n(n-1)(n-2)....2.1(x-1)^{n-1} \text{or}$
 $z_{n} = n! [(x-1)^{n}]_{x} x!$
Putting x =1 in eq. 1 all the terms in RHS become zero except the 1
term which becomes $n!(1+1)^{n} = n!2^{n}$

$$p_n(1) = 1$$
 by the def of $p_n(x)$
 $1 = \text{kn}:2^n$
 $k = \frac{1}{n!2^n}$
 $p_n(x) = ku_n$

$$p_{n}(x) = \frac{1}{2^{n}n!} [(x^{2} - 1)^{n}]$$

$$p_{n}(x) = \frac{1}{2^{n}n!d^{x}} (x^{2} - 1)^{n}$$

Q4. A coin is tossed. If it turns up H, two balls will be drawn from urn A otherwise 2 balls will be drawn from urn B. urn A contains 3 red and 5 blue balls , urn B contains 7 red and 5 blue balls. What is the probability that urn A is used , given that both balls and blue? (find in both cases, when balls were chosen with replacement and without replacement).

Sol- let us define the following events

 A_1 = urn A is chosen

 $A_2 = \text{urn } B$ is chosen

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E= two blue balls are drawn (with reputation)

Then we have

$$p(A_1), \frac{1}{2}$$
$$p(E/A) = \frac{5}{8} *$$

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$$p(A_{2}).\frac{1}{2}$$

$$p(E/A) = \frac{5}{12} * \frac{5}{12}$$

$$\frac{25}{144}$$
So,
$$\frac{p(A/E) = \frac{\left(P(A_{1}) \otimes P(E/A_{1})\right)}{P(A_{1})P(E/A_{1}) + P(A_{2})P(E/A_{2})}$$

$$\frac{\left(\frac{1}{2} * \frac{25}{64}\right)}{\left(\frac{1}{2} * \frac{25}{64} + \frac{1}{2} * \frac{25}{144}\right)}$$

$$\frac{25}{128} + \frac{25}{228}$$

$$\frac{25}{128} + \frac{25}{288}$$

$$\frac{25}{128} + \frac{25}{1152}$$

$$\frac{9}{13}$$
(b) for event $A_{1.}A_{2.}A_{3.}A_{4.}A_{5}...A_{n}$

$$P(U^{n}i = 1A_{i}) \ge \sum_{i=1}^{n} p(A_{i}) - (n-1)$$
Prove that

i.
$$p(\bigcap^{n} i = 1) \ge 1 - \sum_{i=1}^{n} p(\overline{A}_{i})$$
$$p(\bigcap^{n} i = 1A_{i}) \ge 1 - \sum_{i=1}^{n} p(\overline{A}_{i})$$

Q5. State and prove bayes theorem.

Sol. -it states that "If $A_1, A_2 - - - A_N$ are n mutually exclusive event with $P(A_1) \neq 0, i = 1, 2 - - - n$ & B is any other event which can occurred with A or A_1 or A_N then we have,

 $P(A_i/B) = \frac{\left[P(A_i)P(B/A_i)\right]}{\sum_{i=1}^{n} p(A_i)P(B/A_i)}$

Proof- by compound theorem of probability ,

We get

 $P(A;\cap B) = P(A_i).P(B/A;)$

Or $P(A; \cap B) = P(B).P(A; B)$

Given that , B is any other event which occur with A or A; or $\dots A_N$ i.e

$$B = B \cap (A_1 \text{ or } A_t \text{ or } - - - A_n)$$
$$B \cap \left[A_1 \cup A_2 \bigcup - - A_N \right]$$
$$(B \cap A_1) \cup (B \cap A_2) \bigcup - - - U(B \cap A_N)$$

$$P\left[B \cap A_{1} \cup (B \cap A_{2}) \bigcup - - - P(B \cap A_{n})\right]$$

$$p(B \cap A_{1}) + P(B \cap A_{2}) \pm - - P(B \cap A_{n})$$

$$\sum_{i=1}^{n} p(B \cap A_{i})$$

$$\sum_{i=1}^{n} p(A_{i}) P(B'A_{i})$$
Again from II

$$P(A;/B) = \frac{[P(A;)P(B/A;)]}{\sum_{i=1}^{n} p(A;)P(B/A;)}$$

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(b) a random variable X follows binominal distribution with parameter n=40 and $p = \frac{1}{4}$ use chebyshev's inequality to find bounds for. a. p(|X - 10| < 8)

b. p(|X - 10| < 10)



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